Beta-Gamma Directional Correlation Measurement in As^{76}

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The β-γ directional correlation of the first forbidden nonunique $2^- \rightarrow 2^+$ β transition of As^{76} was measured at ten β energies ranging from 1.33 to 2.18 MeV. The angular correlation coefficient $A_2(W)$ varies from 0.076 ± 0.005 to 0.106 ± 0.005 in this energy range. The modified B_{ij} approximation was used to investigate the structure of the first excited state in Se^{76}. A configuration of either $(f_{5/2})^{\pi}(g_{9/2})^2$ or $(f_{5/2})^4(g_{9/2})^{\nu}$ is in agreement with the experimental data. In the first case the $g_{9/2}$ shell would have 2 neutrons with seniority 2, while an even number of π protons with seniority 0 are in the $f_{5/2}$ shell. The other possibility would mean 4 protons in the $f_{5/2}$ shell with seniority 2 and an even number of ν neutrons with seniority 0 in the $g_{9/2}$ shell. The values $-4.2 \leqslant V \leqslant -3.2$ and $-2.8 \leqslant Y \leqslant -2.2$ for the nuclear matrix-element parameters are consistent with experimental data.

INTRODUCTION

In recent years numerous experiments in β decay have helped to establish the fundamental properties of the β-decay interaction.[1] Many theoretical papers have been written giving explicit expressions of the various observables associated with β decay so that experimental results may be readily interpreted.[2,3] Thus β-γ angular correlation measurements in first forbidden β transitions can be used to study nuclear structure. However, the fact that up to six nuclear matrix elements may contribute often makes a quantitative analysis difficult.

Of interest are transitions from a 2^- ground state in odd-odd nuclei to the first excited 2^+ state in even-even nuclei, followed by an electric quadrupole γ transition to the 0^+ ground state. The low-lying 2^+ states in even-even nuclei can be explained by several nuclear models. The strong coupling model[4] assumes rotational excitation, while particle excitation prevails in the j-j coupling shell model.[5] By studying the β decay of odd-odd nuclei with 2^- ground states, it becomes possible to obtain information as to what kind of excitation occurs in the first excited 2^+ state of the daughter nuclei.

If either the K selection rule of the rotational model or the j selection rule of the shell model is operative, all first forbidden matrix elements other than the tensor type B_{ij} matrix element are reduced in magnitude. The fact that the j selection rule is not strict can be explained by assuming configuration mixing. If the B_{ij} matrix element is predominant, a large ft value and a large β-γ anisotropy together with a nonstatistical spectrum shape is expected. However, almost all first forbidden $2^- \rightarrow 2^+$ transitions exhibit an allowed shape, which means the shape factor is energy independent, at least within the accuracy of the measurements. Also a relatively small β-γ anisotropy is found. This behavior indicates that the matrix elements of tensor rank zero and one contribute considerably. A very useful way to describe the situation is the modified B_{ij} approximation.[6,7] Here only the leading terms from the momentum type matrices and the Coulomb corrections for the coordinate type matrices, together with the leading B_{ij} terms, are considered. A limiting case of the approximation is the large Coulomb energy approximation or ξ approximation.[8] Here ξ denotes the Coulomb factor $(\alpha Z)/(2\rho)$, where α is the fine structure constant, Z is the nuclear charge, and ρ is the nuclear radius in units of the electron Compton wavelength.

Let us consider a $2^-(\beta_1)0^+$ and $2^-(\beta_2)2^+$ decay in an odd-odd nucleus [see Fig. 1(a)]. The ground-state decay β_1 is unique with only one matrix element

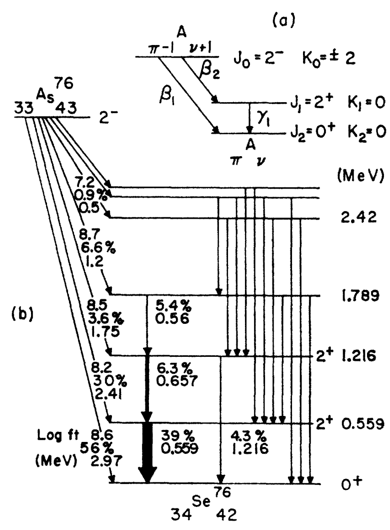

FIG. 1. Decay scheme of As^{76}.

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[1] E. J. Konopinski, Ann. Rev. Nucl. Sci. **9**, 99 (1959).
[2] M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958).
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[4] A. Bohr and M. Mottelson, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Chap. 17, p. 468.
[5] M. G. Mayer and H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1955).
[6] Z. Matumoto, M. Morita, and M. Yamada, Bull. Kobayashi Inst. Phys. Res. **5**, 210 (1955).
[7] Z. Matumoto, M. Yamada, I. T. Wang, and M. Morita, Phys. Rev. **129**, 1308 (1963).
[8] T. Kotani and M. Ross, Phys. Rev. Letters **1**, 140 (1958).

$(\int B_{ij})_1$. In the β_2 decay to the first excited 2^+ state of the daughter nucleus the $(\int B_{ij})_2$ matrix element may be large compared to the others, due to selection rules. If the information obtained from a β_2-γ_1 directional correlation measurement, together with the branching ratio of β_1 and β_2, is combined with the theoretically calculated ratio $(\int B_{ij})_1/(\int B_{ij})_2$ based on a particular nuclear model, it is possible to distinguish between a K selection rule and a j selection rule.[6,7] This gives an indication as to what model should be used to describe the 2^+ state. If it is found that the j selection rule holds, the configuration of the 2^+ state can be investigated. Of course, the method loses some of its power if the contribution from the B_{ij} term is small but should still be quite useful for qualitative insight.

In this paper we will discuss the results of a β-γ directional correlation measurement between the 2.41-MeV ($W_0=5.71$) β group in As^{76} and the successive 0.559-MeV γ transition in Se^{76}. The decay scheme according to the most recent Nuclear Data Sheets[9] is shown in Fig. 1(b). The 26-h As^{76} decay has been investigated quite extensively.[10–17] The shape factor of the 2.41-MeV first inner group was found to be energy independent above 1.7 MeV.[11] The accuracy, however, is limited due to the presence of the other β groups. The β-γ directional correlation coefficient has been reported by Ridgeway and Pipkin[16] at 1.4 and 2 MeV and later by Rose[17] at 1.4, 1.7, and 2.0 MeV. In both measurements the anisotropy was found to be small ($\approx 7\%$) with an uncertainty of 40%.

In order to make an analysis of the 2^+ first excited state in Se^{76}, more accurate data were necessary. Therefore, the β-γ directional correlation coefficient was measured at ten β energies ranging from 1.33 to 2.18 MeV with an accuracy of about 5%. The anisotropy was found to be somewhat larger than expected on the basis of the ξ approximation.[8] An analysis in the modified B_{ij} approximation[7] was made which allowed an examination of the configuration of the first excited state in Se^{76}.

THEORY

The six nuclear matrix elements which contribute to a $2^- \rightarrow 2^+$ first forbidden β transition can be combined into two parameters V and Y representing the relative contributions from the matrix elements with tensor rank zero and one, compared with the matrix element $\int B_{ij}$ of tensor rank two[18]:

$$V=r/z=\left(\xi\int \boldsymbol{\sigma}\cdot\mathbf{r}+\int i\gamma_5\right)\Big/\int B_{ij},$$

$$Y=-s/z=\left(\xi C_V\int \mathbf{r}-\xi C_A\int i\boldsymbol{\sigma}\times\mathbf{r}+C_V\int i\boldsymbol{\alpha}\right)\Big/ C_A\int B_{ij}, \quad (1)$$

where

$$z=\int B_{ij}\Big/\int i\boldsymbol{\sigma}\times\mathbf{r}. \quad (2)$$

In the modified B_{ij} approximation,[6,7] it is assumed that selection rules on the matrix elements of rank zero and one are effective to some degree. Thus, only square terms of r, s, and z are kept in the general expressions given by Morita.[2,7] If it is assumed that time reversal invariance holds, and that the interaction is VA with a two-component neutrino, the following simple expressions for the reduced angular correlation coefficient ϵ_0 and the shape factor C for a $2^-(\beta)2^+(\gamma)0^+$ cascade are obtained[7]:

$$\epsilon_0=[(1/2)(1/14)^{1/2}Y-(1/21)^{1/2}V-(1/112)W]C^{-1}, \quad (3)$$

$$C=Y^2+V^2+(1/12)[p^2+(W_0-W)^2]. \quad (4)$$

The coefficient ϵ_0 is also defined in the following expression for the β-γ directional correlation:

$$N(W,\theta)=1+A_2P_2(\cos\theta)=1+\epsilon_0(p^2/W)[\tfrac{3}{2}\cos^2\theta-\tfrac{1}{2}], \quad (5)$$

where W and p are the relativistic electron energy and momentum. The same expressions can be obtained from reference 3, if only square terms of V, Y, and z are kept. The Coulomb corrections, however, will be neglected here ($\lambda_i=1$) since this correction will become appreciable only for low electron energies and large nuclear charge. It should be noted that for large Y and V the energy dependence of ϵ_0 and C may become negligible and the decay will have the characteristic features of the ξ approximation, namely a constant shape factor and an angular correlation coefficient with a p^2/W energy dependence.

For convenient analysis, Eq. (3) can be written in the form

$$(Y-Y_0)^2+(V-V_0)^2=R^2, \quad (6)$$

where

$$Y_0=0.0668\epsilon_0^{-1}, \quad V_0=-0.109\epsilon_0^{-1} \quad (7)$$

and

$$R^2=Y_0^2+V_0^2-(1/12)[p^2+(W_0-W)^2]-(112\epsilon_0)^{-1}W. \quad (8)$$

[9] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington, D. C.), NRC 59-5-36.
[10] G. Backstrom and I. Marklund, Arkiv Fysik **17**, 393 (1960); R. K. Girgis, R. A. Ricci, and R. Van Lieshout, Nucl. Phys. **13**, 461 (1959).
[11] A. V. Pohm, R. C. Waddell, and E. N. Jensen, Phys. Rev. **101**, 1315 (1956).
[12] M. Spighel, Ann. Phys. (N.Y.) **6**, 535 (1961).
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[14] M. Delabaye, J. P. Deutsch, and P. Lipnik, J. Phys. Radium **23**, 257 (1962).
[15] F. P. Pipkin, G. E. Bradley, and R. E. Simpson, Nucl. Phys. **27**, 353 (1961).
[16] S. L. Ridgeway and F. P. Pipkin, Phys. Rev. **87**, 202 (1952).
[17] H. Rose, Phil. Mag. **44**, 739 (1953).

[18] V corresponds to the parameter X in Morita's notation. The sign of the matrix element $\int i\boldsymbol{\alpha}$ is the same as in reference 2 and most other papers, but opposite from that used by Kotani in reference 3.

Equation (6) describes a circle in the V-Y plane whose radius and center depends upon the experimentally observed coefficient ϵ_0. Following Morita[7] we will call this the experimental circle. In principle the parameter V and Y can now be determined from a second experiment, where some other observable which depends on V and Y (the β-γ circular polarization coefficient for instance) has been measured as a function of energy. The values for V and Y obtained could then be compared with those calculated from the theory. But such a calculation requires a more precise knowledge of the radial part of the nuclear wave functions. For this reason an analysis was suggested by Morita and others[6,7] which uses the ratio of the B_{ij} matrix element of the ground-state β_1 decay and the B_{ij} matrix element of the β_2 decay to the first excited state. The B_{ij} matrix-element ratio $(\int B_{ij})_1/(\int B_{ij})_2$ can be calculated without the knowledge of the radial wave functions, but depends upon the particular nuclear model assumed.

Since the ft value is related to the B_{ij} matrix element by $ft=2\pi^3 \ln 2/C_A{}^2 \sum|B_{ij}|^2$, the B_{ij} matrix-element ratio is inversely proportional to the ratio of the ft values of the β_1 and β_2 decay.

$$(\sum|B_{ij}|^2)_1/(\sum|B_{ij}|^2)_2=(ft)_2/(ft)_1. \quad (9)$$

Here $\sum|B_{ij}|^2=[(2J_1+1)/(2J+1)]|\int B_{ij}|^2$ for a $J\rightarrow J_1$ transition according to the definition of $|\int B_{ij}|^2$ in the notation of Morita.[2,7] It will be convenient to adopt the following notation for the integrated Fermi functions:

$$f=\int_1^{W_0} F(Z,W)C(W)pW(W_0-W)^2dW,$$

$$f_i=\int_1^{W_0} F(Z,W)pW(W_0-W)^2dW, \quad (10)$$

$$f_{ci}=\int_1^{W_0} F(Z,W)(1/12)(p^2+(W_0-W)^2) \times pW(W_0-W)^2dW.$$

The subscript i denotes the respective branch β_i. For the β_1 decay the shape factor is given by $C(W)=(1/12)\times(p^2+(W_0-W)^2)$ with $W_0=6.81$ while for β_2 the shape factor is $C(W)=(1/12)(p^2+(W_0-W)^2)+V^2+Y^2$ with $W_0=5.71$. Making use of Eqs. (10) and observing that the branching ratio $a_i=t/t_i$, where t is the half-life of the initial state and t_i is the half-life of the ith branch, Eq. (9) can be written in the following form:

$$(f_{c2}/f_2)\left\{(a_2f_{c1}/5a_1f_{c2})\left[\left(\int B_{ij}\right)_1\Big/\left(\int B_{ij}\right)_2\right]^2-1\right\}=V^2+Y^2=R'^2. \quad (11)$$

Equation (11) defines a set of theoretical circles, which together with the experimental circles from Eq. (6) can be used to determine the parameters V and Y.

TABLE I. The radii of the theoretical circles $(f_{c2}/f_2)\{(a_2f_{c1}/5a_1f_{c2})\times[(\int B_{ij})_1/(\int B_{ij})_2]^2-1\}=R'^2$. The Roman numbers in column 5 serve to identify the different configurations and their corresponding theoretical circles.

Nuclear model	Configuration of the 2+ state p	n	$\left[\frac{(\int B_{ij})_1}{(\int B_{ij})_2}\right]^2$ [a]	R' [b]	
Protons are excited to seniority 2 and angular momentum 2	$(f_{5/2})^4$	$(g_{9/2})^\nu$	196/5	4.47 4.96	I
	$(f_{5/2})^2$	$(g_{9/2})^\nu$	49/5	1.98 2.25	II
Neutrons are excited to seniority 2 and angular momentum 2	$(f_{5/2})^\pi$	$(g_{9/2})^6$	56/11	1.16 1.39	III
	$(f_{5/2})^\pi$	$(g_{9/2})^4$	126/11	2.20 2.48	IV
	$(f_{5/2})^\pi$	$(g_{9/2})^2$	336/11	3.91 4.34	V
Rotational excitation			7/2	0.70 0.94	VI

[a] Z. Matumoto, M. Yamada, I. T. Wang, and M. Morita, reference 7.
[b] The upper value of R' corresponds to the branching ratio $a_2/a_1=0.54$ (*Nuclear Data Sheets* NRC 59–5–36), while for the lower value the ratio $a_2/a_1=0.66$ was used (Backstrom and Marklund, reference 10). The ratios of the integrated Fermi functions are: $f_{c2}/f_2=1.44$ and $f_{c1}/f_{c2}=3.50$.

The matrix element ratio $(\int B_{ij})_1/(\int B_{ij})_2$ has been calculated for different nuclear models.[6,7] The results for As^{76} are listed in Table I. For the j-j coupling shell model it was assumed that the ground state of the daughter nucleus has seniority zero for both the proton and neutron shell. The first excited state has seniority two, for either the proton or neutron shell, and zero for the other. For proton excitation the B_{ij} matrix-element ratio is given by[6,7]

$$\left(\int B_{ij}\right)_1\Big/\left(\int B_{ij}\right)_2 =(-1)^{j_p-j_n}[\pi(2j_p-1)/10(2j_p+1)(2j_p+1-\pi)]^{1/2} \times[W(22j_pj_p,2j_n)]^{-1}, \quad (12)$$

while for neutron excitation:

$$\left(\int B_{ij}\right)_1\Big/\left(\int B_{ij}\right)_2 =(-1)^{1+j_p-j_n}[(2j_n-1)(2j_n+1-\nu)/10\nu(2j_n+1)]^{1/2} \times[W(22j_nj_n,2j_p)]^{-1}. \quad (13)$$

Note that Eq. (12) is based on the assumption of zero seniority for the neutrons in the 2^+ state and is independent of how many neutrons ν are in the orbit with angular momentum j_n. Similarly Eq. (13) does not depend upon the number of protons π in the j_p orbit.

In Bohr's[4] strong-coupling theory, the ratio $(\int B_{ij})_1/$

$(\int B_{ij})_2$ is independent of the nucleus and given by[6]

$$\left(\int B_{ij}\right)_1 \Big/ \left(\int B_{ij}\right)_2 = [(2J_1+1)/(2J_2+1)]^{1/2} \times (J_0LK_0K_L|J_2K_2)/(J_0LK_0K_L|J_1K_1). \quad (14)$$

The quantum numbers J and K are defined in Fig. 1(a), and L is the rank of the β matrix element. Therefore, in our case $L=J_0=J_1=2$, $K_0=-K_L=\pm 2$, and $J_2=K_1=K_2=0$.

APPARATUS

The β-γ directional correlation measurement was performed with an automated scintillation spectrometer. The γ rays were detected with a 3-in. by 3-in. NaI(Tl) crystal integrally mounted to a Dumont 6363 photomultiplier tube. The unit had a resolution of 7.4% at 0.662 MeV. A NE102 plastic scintillator 0.5 in. thick, coupled via an 0.5-in. tapered lightpipe to a RCA 6342A phototube, was used for detection of the β particles. The crystal was masked along its perimeter with an aluminum shield in order to prevent passage of electrons through the edges and had an effective diameter of 1.3 in. The energy resolution of the Ba^{137} conversion lines was 17%.

The source was mounted on a thin aluminum ring 1-in. diam at the center of a cylindrical vacuum chamber. The chamber had a Lucite lining to reduce electron scattering, and was 5-in. diam and 7 in. high. The distance between the source and the front face of the β and γ crystal was 7 cm and 10 cm, respectively. Lateral shielding of the detectors was found to be unnecessary. The β counter was rigidly mounted to the vacuum chamber, while the γ counter rotated about the outside cylindrical wall. The wall thickness traversed by the γ rays in reaching the NaI(Tl) crystal was 0.32 cm of Lucite and 0.16 cm of aluminum.

The electronic equipment consisted of a "slow-fast" coincidence circuit[19] together with a gated multichannel analyzer for the β energy selection. The risetime of the conventional nonoverload linear amplifier was compensated with a set of time correction circuits.[20] The experiment was performed with a coincidence resolving time of 3.5×10^{-8} sec.

MEASUREMENTS

The As^{76} sources were prepared by irradiating As^{75} evaporated on $\frac{1}{4}$-mil Mylar backings for 7 h in a neutron flux of $2\times10^{12} n/cm^2$-sec in the Ford Nuclear Reactor at the University of Michigan. The diameter of the circular arsenic deposit was 0.9 cm and less than 30 $\mu g/cm^2$ in thickness. In order to prevent contamination of the vacuum chamber due to slow sublimation of the arsenic, the source was covered with a thin layer of zapon ($<20\ \mu g/cm^2$). Gamma-ray spectra of the irradiated Mylar backings revealed the presence of Sb^{122}.[21] However, the Sb^{122} activity was only 1% of the As^{76} activity.

In Fig. 2 a portion of the observed Se^{76} γ spectrum is shown. The γ channel accepted only the 0.559-MeV photopeak. The output of the coincidence circuit controlled the linear gate of the multichannel analyzer. Conversion lines in Ba^{137} and Pb^{207} were used for β-energy calibration. As the L lines could not be resolved, the K/L and $K/(L+M)$ ratios were used to obtain an effective energy of 0.630 and 0.991 MeV. A mercury-wetted relay pulser, coupled to the photomultiplier anode, verified the linearity of the β channel over the entire energy range.

The coincidence counting rate $N(\theta)$ was taken at angles of 90°, 180°, and 270° between the two counters, corrected for accidentals, and normalized to the γ singles counting rate. The average true to chance ratio per β channel was 8:1. The angles were changed every 30 min. The γ-γ coincidence rate was measured by inserting a thin aluminum shield between the source and the β detector. As the β energies in this experiment were well above the energies of the intense γ lines in Se^{76}, the γ-γ coincidence rate was insignificant.

The angular correlation coefficient A_2 was obtained from the measured asymmetry $A=[N(\pi)-N(\pi/2)]/N(\pi/2)$ according to the relation $A_2=2A/Q(3+A)$. The factor Q represents the correction for the finite solid angle subtended by the detectors. The solid angle correction was calculated[22] for the geometry in this experiment and found to be $Q=0.890\pm0.008$. This result was experimentally confirmed using the annihilation radiation of a Na^{22} source. In order to test the accuracy of

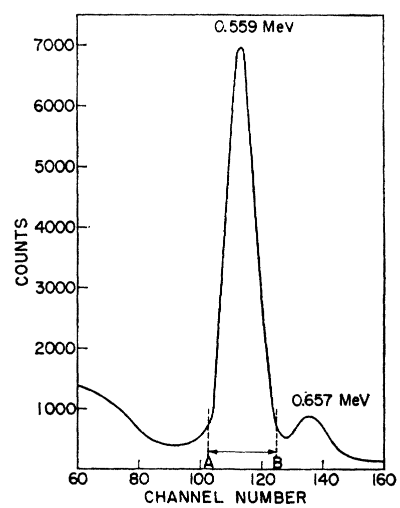

FIG. 2. The relevant portion of the As^{76} γ spectrum. The line AB indicates the γ-channel window.

[19] B. Johansson, Nucl. Instr. **1**, 274 (1957). H. J. Fischbeck and R. G. Wilkinson, Phys. Rev. **120**, 1762 (1960).
[20] J. Draper and A. Fleischer, Rev. Sci. Instr. **31**, 49 (1960).
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[22] H. West, University of California Radiation Laboratory Report UCRL 5451, 1959 (unpublished).

the accidental corrections and the performance of the multichannel analyzer system, the measurement of the asymmetry at 1.88 MeV was repeated with a single-channel analyzer and was well reproduced.

Since the lifetime of the 0.559-MeV state in Se^{76} is 1.2×10^{-11} sec, no attenuation due to extranuclear fields is expected.[9] Nevertheless, a careful check was made by observing the γ-γ angular correlation of the 0.657-MeV transition in coincidence with the 0.559-MeV transition, using a liquid and a solid source. Measurements in 15° intervals under the same conditions for both sources gave the following results for the angular correlation coefficients corrected for finite solid angle:

Liquid: $A_2=-0.157\pm0.007$, $A_4=0.25\pm0.01$,

Solid: $A_2=-0.152\pm0.007$, $A_4=0.26\pm0.01$.

This is in agreement with other measurements reported on this cascade.[23] It should be mentioned that interference effects must be very carefully considered in a measurement of this cascade. By increasing the window width in the channel accepting the 0.559-MeV photopeak, the correlation coefficients were changed drastically and it was possible to reproduce some of the small A_2 coefficients reported earlier.[24]

TABLE II. Summary of the As^{76} β-γ correlation data. The radius R and its center position V_0 and Y_0 is calculated according to Eqs. (6), (7), and (8).

E (MeV)	W (m_0c^2)	A_2	$\epsilon_0=A_2(W/p^2)$	R	$-V_0$	Y_0
1.33	3.60	0.076±0.005	0.0228±0.0015	5.36	4.78	2.93
1.43	3.80	0.078±0.004	0.0221±0.0011	5.52	4.93	3.02
1.52	3.97	0.077±0.004	0.0207±0.0010	5.91	5.26	3.23
1.60	4.13	0.078±0.004	0.0200±0.0010	6.12	5.45	3.34
1.70	4.33	0.087±0.004	0.0212±0.0010	5.74	5.14	3.15
1.80	4.52	0.094±0.004	0.0219±0.0010	5.52	4.98	3.05
1.88	4.68	0.091±0.004	0.0204±0.0010	5.95	5.34	3.27
1.97	4.86	0.101±0.005	0.0219±0.0010	5.49	4.98	3.05
2.09	5.09	0.099±0.005	0.0202±0.0010	5.98	5.40	3.31
2.18	5.27	0.106±0.005	0.0209±0.0010	5.74	5.22	3.20

The effects of backscattering and the finite energy resolution of the β-scintillation spectrometer were extensively investigated. The β detector was mounted at the focus of a thin lens magnetic spectrometer set at a momentum resolution of 2%. A multichannel analyzer was used to record the response of the β detector to the nearly monoenergetic electrons from the spectrometer. The backscatter tails were small and could be well represented by straight almost horizontal lines, which had a mean height of (1±0.2)% of the peak height. This agrees well with a recent measurement by Bertolini *et al.*[25] It is considerably less, however, than the backscatter tail of 7% for anthracene crystals reported by Freedman *et al.*[26] With the measured response curves the method described in reference 26 was used to obtain the correction for the observed spectrum and for the asymmetry. The resulting corrections were small enough to be neglected within the present accuracy of the data. This is to be expected since the observed energy dependence of the asymmetry is small.

[23] Z. Grabowski, S. Gustafsson, and I. Marklund, Arkiv Fysik **17**, 411 (1960).
[24] E. G. Funk and M. L. Wiedenbeck, Phys. Rev. **109**, 922 (1958).
[25] G. Bertolini, F. Cappelani, and A. Rota, Nucl. Instr. Methods **9**, 107 (1960).

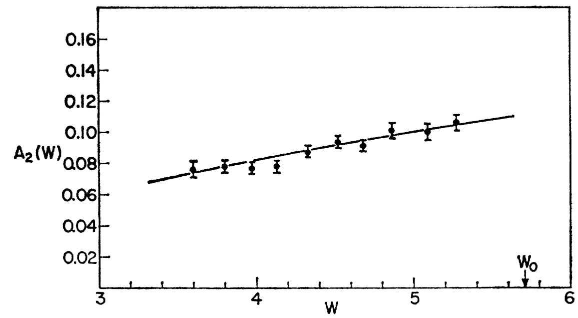

FIG. 3. Energy dependence (m_0c^2 units) of the correlation coefficient A_2. The solid curve is a theoretical one with $Y=-2.3$ and $V=-3.4$.

A correction for the interference of the 1.75-MeV second inner group was applied to the low-energy points. In order to make this correction, the asymmetry of the 1.216-MeV γ transition in coincidence with the 1.75-MeV β group was measured in the energy range from 1.2 to 1.5 MeV. The result corrected for accidentals and finite solid angle is $A=+0.001\pm0.007$ which is isotropic within the experimental error. If the interfering correlation is assumed to be isotropic, the true asymmetry A is found from the observed asymmetry A' by $A=A'[1+(N_i(270°)/N_t(270°)]$. The ratio $N_i(270°)/N_t(270°)$ represents the coincidence contribution from the interfering correlation to that of the correlation under study. This ratio was measured for the lowest energy points by accumulating the gated γ spectrum instead of the gated β spectrum in the multichannel analyzer, and is given below:

E(MeV)	1.33	1.43	1.52
$N_i(270°)/N_t(270°)$	0.10	0.06	0.02

ANALYSIS OF DATA

The β-γ directional-correlation data for the 2.41-MeV ($W_0=5.71$) first inner β group in As^{76} are summarized in Table II. The energy dependence of the correlation coefficient A_2 shown in Fig. 3 ranges from +0.076 at 1.33 MeV to 0.106 at 2.18 MeV. The low-energy points were corrected for contributions from the 1.75-MeV ($W_0=4.4$) β group. A graphical analysis has been employed to obtain probable values for V and Y.

[26] M. S. Freedman, T. B. Novey, F. T. Porter, and F. Wagner, Jr., Rev. Sci. Instr. **27**, 716 (1956).

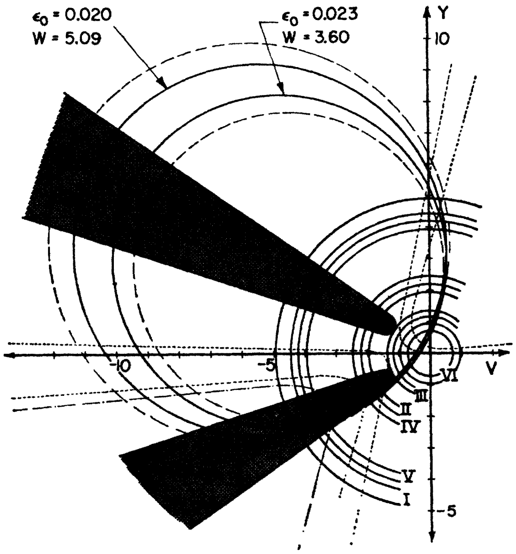

FIG. 4. Experimental results analyzed in the modified B_{ij} approximation together with the theoretical circles (I to VI) shown in the V, Y parameter plane. The shaded area represents the results of the dynamical orientation correlation by Pipkin *et al.*[15] The area between the dotted hyperbolas indicates the result of the circular polarization correlation by Delabaye *et al.*,[14] at an average energy $W=3$ and an average angle of 150°. The hyperbola drawn as alternating dashes and dots was calculated for $W=5$ in order to indicate the possible energy spread.

As can be seen from Eq. (6), the measured reduced correlation coefficient ϵ_0 determines a circle of possible V and Y values in the V, Y plane for each energy. The radii of the experimental circles and their center position have been calculated using Eqs. (7) and (8), and are listed in columns 5, 6, and 7 of Table II. In Fig. 4, the V, Y plane is shown. However, only the circles for the two energies $W=3.60$ and $W=5.09$ are drawn. The circles which correspond to the other energies for which ϵ_0 was measured all lie between those two circles. Because of the experimental uncertainty, each circle is broadened. Therefore, all points (V,Y) which lie in the area between the error limits (dashed circles) of the two shown experimental circles agree with the β-γ directional correlation. In order to specify particular values for V and Y, additional information is necessary. This information is obtained from the theoretical circles and other experiments which also define an area of points (V,Y) in the V, Y plane. If it is found that the experimental curves intersect with theoretical circles, the corresponding (V,Y) pair will be assumed probable. The β-γ correlation coefficient and the shape factor are then calculated using the probable V and Y values, and compared with the experimentally observed β-γ directional correlation and shape factor. The results from the β-γ circular polarization correlation and from the angular distribution of the γ radiation from oriented As^{76} nuclei have been included in the analysis and are briefly described below.

The angular distribution of the 0.559-MeV γ radiation from dynamically oriented As^{76} nuclei was observed by Pipkin *et al.*[15] The distribution is given by the expression:

$$W(\theta)=1-(10/7)g_2B_2P_2(\cos\theta)-(40/3)g_4B_4P_4(\cos\theta). \quad (15)$$

The parameters g_i describe the orientation of the As^{76} ground state. B_2 and B_4 depend upon the β radiation emitted previous to the emission of the γ ray and are given as

$$B_2=\alpha_0+(1/2)\alpha_1-(3/13)\alpha_2,$$
$$B_4=\alpha_0-(2/3)\alpha_1+(2/7)\alpha_2. \quad (16)$$

The α_L in Eq. (16) are the fraction of decays which occur with the emission of L units of angular momentum.

$$\alpha_L=f_c(L)/\textstyle\sum_L f_c(L), \quad (17)$$

where $L=0$, 1, 2, and $\sum_L \alpha_L=1$. The integrated Fermi functions $f_c(L)$ are defined as

$$f_c(0)=V^2\int F(Z,W)pW(W_0-W)^2dW,$$

$$f_c(1)=Y^2\int F(Z,W)pW(W_0-W)^2dW,$$

$$f_c(2)=\int F(Z,W)pW(W_0-W)^2\times(1/12)(p^2+(W_0-W)^2)dW.$$

From their measured ratio $B_2/B_4=1.5\pm0.3$ and from $B_4\eta=0.30\pm0.03$, where η $(0.3\leq\eta\leq1)$ represents an efficiency of orientation, these authors conclude that: $0.1\leq\alpha_1\leq0.3$ and $\leq0\alpha_2\leq0.45$. The domain in the V, Y plane defined by those results is shown in Fig. 4 as the shaded area.

The β-circularly polarized γ correlation,

$$N(W,\theta)=1+\omega(p/W)\cos\theta, \quad (18)$$

has been remeasured recently by Delabaye *et al.*[14] Their result is $\omega=-0.035\pm0.049$ for β energies of $W>1.7\times(m_0c^2)$ and at an instrument angle of 180°. If we assume that the average energy was $W=3$ and that the average angle between the β and γ rays was 150°, an analysis in the modified B_{ij} approximation is possible. However, this may not be quite realistic as the correlation was measured on an integral basis and the energy dependence may not be negligible (Fig. 8). In the modified B_{ij} approximation for a $2^-(\beta)2^+(\gamma)0^+$ transition ω is given as[7]

$$\omega=(-0.816VY+0.167Y^2)/(V^2+Y^2)\left[\frac{-0.312YW-p^2(0.073+0.143\cos^2\theta)+0.021q^2}{(p^2/W)(0.134Y-0.218V-0.009W)P_2(\cos\theta)+(1/12)(p^2+q^2)}\right]. \quad (19)$$

In the above expression $q^2=(W_0-W)^2$. The results assuming $W=3$ and $\theta=150°$ are shown as dotted hyperbolas in Fig. 4. In order to indicate the possible energy spread, the curve obtained from Eq. (19) with the lower limit for ω ($\omega=-0.084$) and $W=5$ has been included in Fig. 4.

The circles centered around the origin of the V, Y plane in Fig. 4 and numbered I to VI represent the theoretical circles. The radii have been calculated according to Eq. (11) and are listed in Table I, together with the nuclear model upon which the calculation of the nuclear matrix element ratio was based. The broadening of the theoretical circles is caused by the experimental uncertainty of the measured branching ratios. The two values used for the branching ratios are $a_2/a_1=30.6/56.1$ from reference 9 and $a_2/a_1=35/53$ from reference 10.

From the overlapping area of the different curves in Fig. 4 probable values for V and Y can be chosen.

TABLE III. The probable values of the nuclear matrix element parameters Y and V. Column 4 refers to Table I. Column 5 indicates fit with the observed coefficient ϵ_0 as shown in Fig. 6.

Y	V	Overlapping area in V-Y plane: Experiment[a]	Theoretical circle	Fit
−2.5	−4.0	ϵ_0, orientation	I	good
−2.3	−3.4	ϵ_0, orientation	V	good
−1.5	−1.8	ϵ_0, ω, orientation	IV	fair
−1.3	−1.6	ϵ_0, ω, orientation	II	poor
−0 9	−1.0	ϵ_0, ω	III	no
−0.5	−0.8	ϵ_0	VI	no
+2.5	+0.5	ϵ_0	IV	poor
+4.9	+0.5	ϵ_0	V	poor
+5.0	+0.2	ϵ_0, ω	I	poor

[a] "Orientation" indicates the results of the dynamical orientation correlation by Pipkin *et al.*, reference 15. The symbol ω indicates the results of a measurement of the β-γ circular polarization correlation by Delabaye *et al.*, reference 14, analyzed in the modified B_{ij} approximation.

These values are then inserted into the theoretical expression for ϵ_0 and C [Eqs. (3) and (4)] and compared with the experiment. Figure 5 shows the calculated shape factor normalized to one at $W=1$ for the different sets of Y and V listed in Table III. The Roman numbers (column 5 in Table I and column 4 in Table III) serve to identify the theoretical circles and the corresponding sets of Y and V. In Fig. 6 a comparison of the possible pairs of V and Y with the observed correlation coefficient ϵ_0 is shown. Column 5 in Table III indicates how well the particular values for V and Y fit the experimental results. The V, Y pair obtained from the overlap of the β-γ directional correlation with the results of the dynamical orientation and with theoretical circle I and V fits an energy independent shape factor and the reduced correlation coefficient ϵ_0 best. The corresponding range of V and Y in Fig. 4 is $-4.2\leq V\leq-3.2$ and $-2.8\leq Y\leq-2.2$.

Since the expressions of the modified B_{ij} approximation reduce to the ξ approximation, an analysis in this approximation should lead to V and Y values compatible with the above results, if the energy dependence of ϵ_0 is small. The expressions in the ξ approximation can be obtained from Eqs. (3), (4), (8) and (19) by neglecting all energy-dependent terms. To be consistent it is also assumed that the fraction of decays which occur with the emission of two units of angular momentum α_2 in reference 15 can be neglected. The ratio Y/V is then obtained directly from the measured ratio $B_2/B_4=1.5\pm0.3$. Another restriction on V and Y is obtained from the β-γ resonance-fluorescence correlation[12]:

$$N(\theta)\approx1-\alpha(p/Wq)\cos\theta(E_\gamma+p\cos\theta), \qquad (20)$$

where $\alpha=1-(5/3)Y^2/(V^2+Y^2)$ was found to be within $-0.2\leq\alpha\leq0.9$. All experimental results, analyzed in the ξ approximation, are plotted in Fig. 7. The average value $\epsilon_0=0.0211\pm0.004$ was used for the reduced correlation coefficient in Eq. (6). As can be seen from Fig. 7, the results of the circular polarization correlation do not agree with the dynamical orientation correlation in this approximation. From the overlap of the curves for ϵ_0, B_2/B_4, and α it can be seen that the values $-4.2\leq V\leq-3.2$ and $-2.8\leq Y\leq-2.2$ are also in agreement with the ξ approximation. The possibility

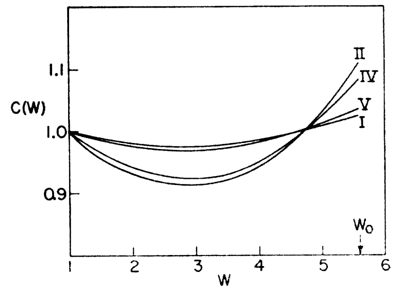

FIG. 5. The shape factor calculated in the modified B_{ij} approximation for various sets of V and Y given in Table III.

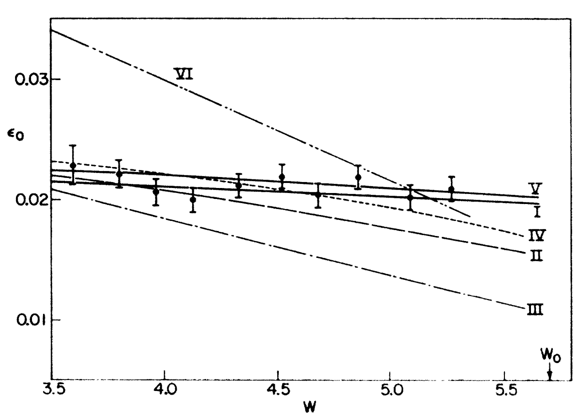

FIG. 6. Comparison of the theoretical "reduced correlation coefficient" ϵ_0 [Eq. (3)] for the possible sets of V and Y from Table III with the measured values ϵ_0.

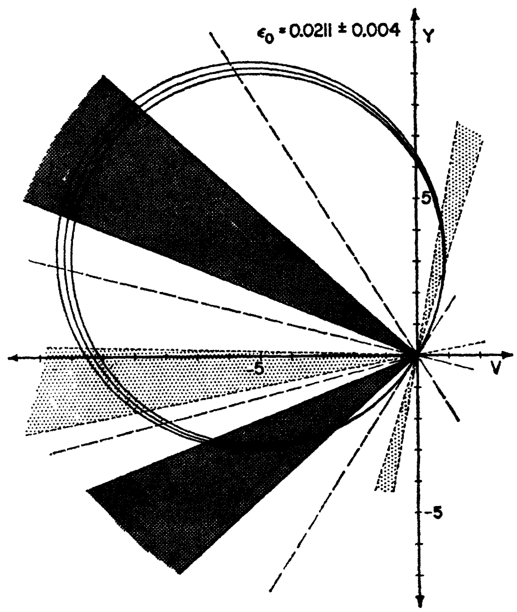

FIG. 7. Experimental results analyzed in the ξ approximation. The heavy shaded area represents the results of the dynamical orientation correlation.[15] The lightly shaded area gives the results of the circular polarization correlation.[14] The area between the dashed lines in the second and third quadrant indicates the results of the resonance fluorescence correlation.[12]

$V<-8$ and $Y>4$ has no corresponding theoretical circle and will, therefore, not be considered here.

CONCLUSION

From the results of the β-γ directional-correlation measurements and the analysis in the modified B_{ij} approximation, rotational excitation of the first excited 2^+ state in Se^{76} can be ruled out. (Table III and Fig. 6). Two possible configurations are found to be in close agreement with the experimental data. One possibility is the configuration $(f_{5/2})^{\pi}(g_{9/2})^2$, where the $f_{5/2}$ shell has an even number of π protons with seniority zero and the $g_{9/2}$ shell has two neutrons with seniority two and angular momentum two. The ground state of Se^{76} would then be $(f_{5/2})^{\pi}(g_{9/2})^2$ with seniority zero for both proton and neutron shell while the ground state of As^{76} would be $(f_{5/2})^{\pi-1}(g_{9/2})^3$. However, the configuration $(f_{5/2})^4(g_{9/2})^{\nu}$ for the 0.559-MeV 2^+ state in Se^{76} with 4 protons excited to have seniority two and angular momentum two, is also in good agreement with the present experimental information. The two nuclear parameters consistent with both configurations are $-4.2\leq V\leq -3.2$ and $-2.8\leq Y\leq -2.2$. A comparison of the above values for V and Y with the experimental results is shown in Fig. 3. The solid line was calculated in the modified B_{ij} approximation with $V=-3.4$ and $Y=-2.3$. The upper limits of the contribution from the B_{ij} matrix element to the 2.41-MeV β transition in As^{76} are

$$\left|C_A\int B_{ij}\right|<0.3\left|\xi C_A\int \boldsymbol{\sigma}\cdot\mathbf{r}+C_A\int i\gamma_5\right|,$$
$$\left|C_A\int B_{ij}\right|<0.5\left|\xi C_A\int i\boldsymbol{\sigma}\times\mathbf{r}+\xi C_V\int \mathbf{r}+C_V\int i\boldsymbol{\alpha}\right|. \quad (21)$$

This result indicates that the application of the modified B_{ij} approximation has to be considered with caution. Especially since the end-point energy of the β spectrum is $W_0=5.71$, while the Coulomb factor $\xi=9.4$. On the other hand, the ft value for the 2.41-MeV transition is $\log(ft)=8.2$ compared to $\log(ft)=8.6$ for the pure B_{ij} groundstate transition. We, therefore, assumed the B_{ij} approximation with the restrictions mentioned in reference 7. However, it is reassuring that the results of this approximation are in agreement with the most simple configuration which can be assumed on the basis of the shell model for the first excited state in Se^{76}.

Since it is not possible at present to decide unambiguously between the $(f_{5/2})^{\pi}(g_{9/2})^2$ and $(f_{5/2})^4(g_{9/2})^{\nu}$ configuration for the first excited state in Se^{76}, improved measurements of other observables are suggested. Especially useful would be a measurement of the circular polarization correlation as a function of energy and angle. (Compare Fig. 8.) Also conclusive evidence as to whether the shape factor is energy independent would be desirable. However, this may be difficult because of the limited energy range which can be investigated.

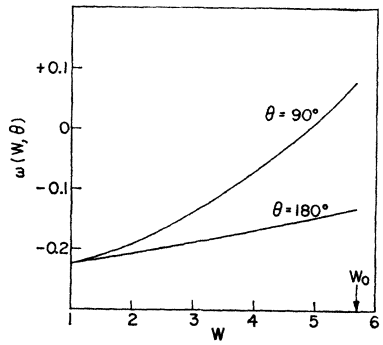

FIG. 8. The circular polarization coefficient as a function of energy (m_0c^2 units) and angle, calculated in the modified B_{ij} approximation with $Y=-2.3$ and $V=-3.4$.

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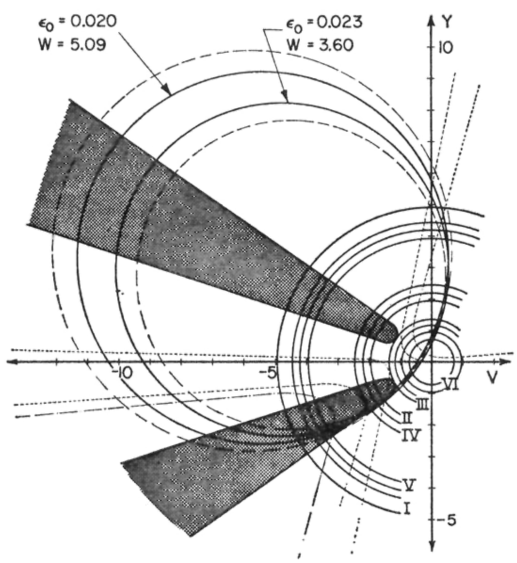

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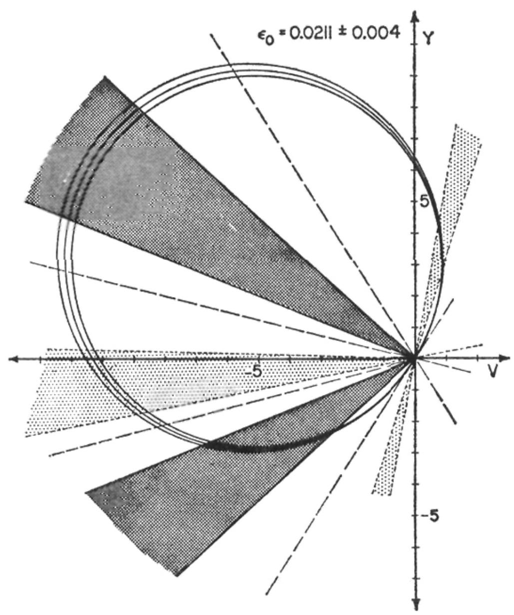

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